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Office 519

Number theory

Fall Semester 2014

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MIDTERM

Maximal Score: 230 points

The grader cannot be expected to work his way through a sprawling mess of identities presented without a coherent narrative through line. If he can't make sense of it in finite time you could lose serious points. Coherent, readable exposition of your work is half the job in mathematics. The amount of points for the given question might give you an idea about how short the solution I expect is. You should have enough space to answer to the question in the space I leave to you (if not write on the back). Be careful: THERE IS 5 PAGES. Do not forget to write you name.
NO MATERIAL ALLOWED AS WELL AS CALCULATORS!

Problem 1: 20 points Find the remainder when 2^{33} is divided by 31.

Problem 2: 40 points Suppose that $n^2 = \sum_{d|n} f(d)$. Evaluate $f(8)$.

Problem 3: 20 points
Prove that $n - 1$ and $2n - 1$ are relatively prime, for all integers $n > 1$.

Problem 4: 40 points
Find all solutions to the congruence $x^2 \equiv p \ mod \ p^2$ when p is a prime number. (Hint: consider mod p).

Problem 5: 30 points
Show that n^4+n^2+1 is composite for all $n \geq 2$. (Hint: $n^4+n^2+1 = (n^4+2n^2+1)-n^2$.)

Problem 6: 60 points Give all the solution:

1. Solve the system of congruences $5x \equiv 14 \ (mod\ 17)$ and $3x \equiv 2 \ (mod\ 13)$.
2. Solve the congruences $5x \equiv 2 \ (mod\ 10)$.

Problem 7: 20 points Prove that $10^{n+1} + 4 \times 10^n + 4$ is divisible by 9, for all positive integers n (Hint: Think!)